ACME Advisory Committee on Mathematics Education

National Curriculum Review 2011

About ACME

The Advisory Committee on Mathematics Education (ACME) is an independent committee, based at the Royal Society and operating under its auspices, that aims to influence Government strategy and policies with a view to improving the outcomes of mathematics teaching and learning in England and so secure a mathematically enabled population.

This submission to the National Curriculum review has been prepared by members of ACME in consultation with members of the ACME 'Outer Circle', who provide a breadth of experience for ACME to call on in developing its positions.

Our conclusions are heavily informed by our work on the 'Mathematical Needs' project (see below for details).

Headline messages

- The new National Curriculum should:
    - seek to raise the standards of mathematical knowledge, in order to support the next generation as they live and work in an increasingly mathematical world.
    - be based on a clear understanding of the various outcomes which it is designed to foster in order to provide a rationale for its content.
    - ensure that mathematical thinking is developed, including problem-solving, modelling, applications, reasoning, generalisation, proof and classification.
    - show the sophisticated connections and relationships between key mathematical ideas in a non-linear fashion. It must provide sufficient examples in accompanying documentation to avoid misinterpretation. There must also be coherence across all subjects.
    - present the twin aspects of 'methods in mathematics' and 'applications of mathematics' in tandem, reflecting the philosophy of the linked pair of GCSEs in mathematics currently being piloted.
    - be a catalyst for a move towards many more people continuing to study some form of mathematics post-16, in anticipation of all young people being required to continue in education or training up to the age of 18.
- A new National Curriculum is not enough on its own to improve standards – accompanying assessment at any level must be appropriate, and many other control factors should be considered alongside the review including inspection, pedagogy and professional development.[1]
- The 1999 National Curriculum for Key Stages 1-4 provides a detailed and carefully thought through mathematics specification and, together with the focus on the accompanying process skills in the 2007 specification for Key Stage 3-4, provides a good basis from which to work.
- Although the intention is that the new National Curriculum will remain silent on pedagogical matters, it is important that it fosters good teaching and learning and ensures that students at all attainment levels will experience a wide range of mathematical ways of working.

[1] A comprehensive list of control factors is given in *Could Do Better: Using International Comparisons to refine the National Curriculum in England* (Tim Oates 2011) – available from http://www.cambridgeassessment.org.uk/ca/digitalAssets/188853_Could_do_better_FINAL_inc_foreword.pdf

Introduction

1. ACME welcomes the decision that mathematics will continue to be part of the National Curriculum, as we believe that mathematics is important as a subject on its own as well as being closely related to other science, engineering and technology subjects and beyond.

2. Over the last two years, ACME has been working on a project looking at the 'mathematical needs' of the nation, both from the perspective of Higher Education and employment and from the learners' point of view. We believe that this work will be highly relevant to the National Curriculum review. Findings from this project form the basis of many aspects of this submission, and it is expected that the full reports will be published in May 2011.

3. Given that many other mathematics organisations and individuals will be responding at a detailed level – and that there is likely to be broad agreement on the general topics that should be included in the National Curriculum – this response focuses on high-level issues.

The purposes of a National Curriculum

4. It is important to reflect on the purposes of a national curriculum before considering the content of such a document.

5. One such purpose is to define what we, as a country, need students to have studied by the end of their compulsory education. No doubt this was considered in the drafting of the original National Curriculum over 20 years ago. However, since then there have been major changes in the way we lead our lives and our understanding of the role and practice of education. It would be surprising if the needs of the country have not changed. Moreover, they will continue to change – it is important therefore that the National Curriculum provides students with the tools they need to adapt to the needs of a world whose nature is difficult to predict.

6. A parallel purpose of a national curriculum is to support good teaching and learning by setting out the 'essential knowledge' in such a way as to encourage good pedagogy, even if this pedagogy is not defined. Again, much has changed since the original National Curriculum and major advances have been made in our understanding of the components of effective teaching and learning. We can use this knowledge to ensure that the new National Curriculum is framed in such a way as to encourage a broader approach.

7. Allied to the changes in the 'mathematical needs' of the country over time is an increasing awareness of the inadequacy of our current mathematics provision. At all levels we need more people to have a deeper knowledge of mathematics but England is out of step with other OECD countries in neither requiring nor expecting a large majority of young people to continue mathematics post-16[2]. It is predictable that at some point in the life of the new National Curriculum this situation will change and many more young people will continue mathematics after GCSE. Many of these new students will be a different clientele from those taking AS and A level mathematics and will progress to new courses. This changing situation should have a fundamental impact on this review. The National Curriculum will no longer take most students to the end of a road but, instead, to a point from which they will continue in different directions. This is, to some extent, recognised in the philosophy underpinning the current pilot of the twin GCSE in Mathematics.

[2] Hodgen, J. et al *Is the UK an Outlier? An international comparison of upper secondary education*. The Nuffield Foundation (2010), available from http://www.nuffieldfoundation.org/sites/default/files/files/Is%20the%20UK%20an%20Outlier_Nuffield%20Foundation_v_FINAL.pdf

8. The National Curriculum in Mathematics should therefore provide a basis for the very varied pathways which students will follow post-16[3]. These include A-level mathematics possibly also with AS-or A-level Further Mathematics, continuing NQF level 2 mathematics, and intermediate qualifications such as Use of Mathematics and level 3 FSMQs.

Principles in the process of designing a National Curriculum for Mathematics

9. The required strategic outcomes from the National Curriculum should be agreed, in order to allow any potential changes to be considered against them, and accepted or rejected. This process will lead not just to a list of content and skills, but to a well thought out rationale for it.

10. An expert group of mathematicians and mathematics educators will be needed to consider the evidence which this consultation elicits and build the revised National Curriculum. This group should have expertise across the 5-16 age range, and also considerably wider than this. It must include representation from Higher Education (HE) mathematics to provide an intellectual lead; it must also include representation from a range of other HE subjects to capture the way in which mathematics is now used in other disciplines, and from those with expertise in mathematics in the workplace (in view of the recommendations in the recent Wolf review and ACME's work on Mathematical Needs (to be published in June 2011)).

The Nature of Mathematics

11. Mathematics involves a set of particular ways of understanding and reasoning about real and imagined objects, using numerical, structural and spatial characteristics and relationships. The components of mathematics include:

    - knowing and using facts, methods, conventions, definitions, language and theorems which apply in particular domains
    - being familiar with concepts, and combinations of concepts, which are defined on domains
    - engaging with and understanding relations between concepts
    - using and translating between notations, models and representations of situations within and outside mathematics
    - using appropriate calculational methods to work out the solutions of arithmetic problems
    - manipulating and communicating with symbols and formal expressions using formal rules of combination
    - seeking and using numerical, spatial, algebraic and logical structures, and the properties of mathematical objects
    - exploring and manipulating mathematical ideas and solving a wide range of mathematical problems
    - applications, including mathematical modelling[4]
    - reasoning, such as deducing from axioms, hypothesising, generalising, and proving
    - making abstractions and generalisations

12. These components cannot be taught and learnt separately. The components link together in networks, hierarchies, layers, and other connections. For example, until you understand what a concept denotes, can use its notation, can reason about it, know some associated facts and

[3] *Post-16 in 2016: proposals for 16-19 Mathematics in anticipation of the review of qualifications scheduled for 2013 with resulting changes to be implemented from 2016*. ACME (2010), available from http://acme-uk.org/news/news-items-repository/2010/7/launch-post-16-in-2016-position-paper

[4] 'Mathematical modelling' should be understood to mean the process of describing and analysing real world phenomena in mathematical language

theorems, and can represent it in equivalent ways, you cannot use it to solve complex problems or create concepts that are new to you. On the other hand, to understand the full meaning of a concept, you have to experience it in a range of situations and representations and also have opportunities to explore the possibilities it offers. Abstract, procedural, conceptual and utilitarian aspects have to develop together.

Fundamental features of a National Curriculum for mathematics

13. For the reasons described above, ACME believes that the National Curriculum for mathematics should:

    - Be based on key mathematical ideas and how they are related in complex ways
    - Have sufficient detail and examples to avoid misinterpretation
    - Promote all aspects of mathematical proficiency, (otherwise only what is tested will be taught). These include generic mathematical competencies – such as knowing how to break a complex problem into a series of simpler problems, an ability to talk about mathematics using correct mathematical vocabulary, a fluency with calculation and algebraic manipulation, etc. – and mathematical attitudes, such as motivation to study mathematics, perseverance, and developing the confidence and willingness 'to have a go' in an unfamiliar situation
    - Incorporate the mathematical capabilities, methods and questions which arise from all available technologies, especially those used in the workplace and those that are designed to support the development of mathematical ways of thinking
    - Ensure that mathematical thinking is developed, including problem solving, modelling, reasoning, generalisation, proof, classification
    - Develop awareness of the place of mathematics in our culture (including a historical perspective).
    - Be subject to an agreed and well-defined review cycle – held perhaps every ten years – but with clear stability between reviews. Reviews should be informed by societal needs, advances in mathematics and technology, and the current needs of adolescent learners

14. As a result, the mathematics National Curriculum should encourage[5]:

    - Procedural recall, accuracy and fluency
    - Integrated conceptual and representational understanding
    - Accumulation of a repertoire of useful knowledge
    - Strategic competence in mathematising, hypothesis-testing, and problem-solving (including working with current technology)
    - Mathematical reasoning applied to learning, understanding, using and communicating mathematics
    - Appreciation of the power and structure of mathematics and willingness to use them.

15. The National Curriculum must show the sophisticated connections and relationships between key mathematical ideas in a non-linear fashion, representing cross-curriculum ideas clearly. It

[5] NCETM (2008) *Mathematics Matters*, National Centre for Excellence in the Teaching of Mathematics (NCETM), https://www.ncetm.org.uk/resources/12491,

is important to set out the content in detail, organised to show the interdependencies of the various ideas involved. Specific examples to illustrate the level of sophistication intended at a particular stage are essential. A potential model for communicating mathematical connections to non-specialist teachers is being developed as part of ACME's work on Mathematical Needs – ACME is happy to provide these diagrams in advance of publication in May 2011.

16. The new National Curriculum should maintain a balance between content and the necessary more open-ended skills required to analyse unfamiliar problems and to develop a flexible armoury of mathematical tools to cope with a rapidly changing world.
17. There should be coherence across all National Curriculum subjects, with consistent terminology and links to and from other subject areas which use mathematics to analyse problems (physics, chemistry, biology, geography, economics etc).
18. It is important for students to develop both understanding and reasoning in parallel with a focus on memorisation of facts and manipulation/calculation of numbers. Students need to develop mathematical ways of thinking and working.
19. The National Curriculum should allow the needs of higher attaining pupils to be met through more exposure to deeper problems that develop greater depth and connections in their learning.

Key strengths of the current National Curriculum and opportunities for improving it

20. A key strength of the current National Curriculum is the recognition which it gives to the central role of Mathematics.
21. The current National Curriculum provides continuity across transition, including common background for post-16 mathematics and wider education. Supplementary guidance and progression support via National Strategies materials also supports this.
22. The 2007 National Curriculum for KS3-4 has good intentions in that it sets out to develop reasoning, critical thinking, communication and inquiry skills and thereby develop problem solving skills, but it is vague on the exemplification of the exact mathematical content and its demand. As ACME noted in 2007:

    "The swing from specific to descriptive wording [in the proposals for the 2007 National Curriculum for KS3-4] *means that it will not be clear to teachers, pupils, parents, QCA and Awarding Bodies what mathematics will be taught at Key Stage 3 and 4... [W]hile a detailed curriculum runs the risk of being misconstrued as a list of separate topics,...* [teachers] *are unlikely to do a better job if left to* [interpret] *the vague and sometimes misleading statements."*[6]
23. The new National Curriculum needs more detail than the 2007 version for KS3-4 has, in terms of exemplification and cognitive dependencies if it is to fulfil the purposes described above
24. It is not the intended content of the National Curriculum which is the major fault, but the way in which it is assessed and the high-stakes target culture which has led to 'teaching to the test'. It is not enough for students to simply rote-learn content, since it does not give them a firm foundation on which to build later learning or use in the world of work or in adult life, nor the confidence to do so.

[6] ACME's response to the QCA Secondary Curriculum Review, April 2007, available from http://www.acme-uk.org/media/4741/acme%20response%20to%20qcas%20secondart%20curriculum%20review_%202007.pdf

25. At the end of the day, it may not be the actual specification of the mathematics curriculum that determines the quality of students' learning of mathematics, but the quality of teaching, the available support materials and the quality of inspection and assessment that supports the curriculum[7]. The curriculum should aim high and be aspirational and this should become the driver for the development of appropriate, high quality assessment instruments and resources

Documentation

26. Two layers of statutory documentation should be produced – one for policy-makers and parents that describes the core ideas, and another providing additional guidance for teachers to ensure that the National Curriculum is implemented coherently. The latter would be especially helpful for non-specialist teachers, who may not understand the connections between different mathematics concepts. If the detail is not provided officially it will be provided by awarding bodies and publishers, with the inevitable 'lowest common denominator' effects attending high stakes assessment.

27. ICT may help to provide documentation which reflects the sophistication and complexity of the layers.

28. The structure of the statutory documentation must ensure that the concepts described are taught and learned in depth. Depth of understanding of core concepts is important for progression.

Specifying lesson time

29. Since mathematics is fundamental to intellectual development and progression an indication of the proportion of the total school timetable should be given. As mathematics is one of the core subjects, there is an assumption that the teaching time will be frequent and substantial. We assume that the international norms will be consulted to make sure that the amount of lesson time given to mathematics is appropriate[8]. TIMMS gives comparable data, but 'guidelines' can quickly become *de facto* maxima and this should be taken into account when determining the guidelines.

30. It is important that hours are not reduced either because a student has reached a benchmark standard or because a student is having difficulties and progression is slow with little apparent prospect of reaching a benchmark.

Essential knowledge in the programme of study for mathematics

31. The 1999 National Curriculum for Key Stages 1-4 includes a detailed and carefully thought through mathematics specification, which also involved international comparisons at that time and responded to concerns expressed by working mathematicians in HE and in industry. The topics covered by this version, together with the refocus on the accompanying process skills evident in the 2007 National Curriculum for KS3-4, are generally felt to be a good basis from which to work.

32. Although much of the suitable content for a new National Curriculum can be derived from international comparisons and the 1999 version, newer aspects of mathematics that are crucial for workplace, personal finance and further study (such as statistical reasoning, risk,

[7] See footnote 1.

[8] Relevant information can be found in *Education at a Glance 2010: OECD indicators* (available from http://www.oecd.org/edu/eag2010)

modelling, programming) might not be found that way. Most international curricula do, however, include methods of mathematical work that need to be experienced in school in order to develop flexible, problem-solving, and application approaches to mathematics.

33. There should be an increase of emphasis on the importance of algebraic and arithmetic manipulation, more on the importance of critical reasoning, problem solving and the clear communication of mathematics to other people. This wider experience of mathematics needs to have appropriate forms of assessment otherwise it is unlikely to be delivered adequately; and adequate delivery depends on good pedagogy from teachers and the availability of high-quality resources

34. Knowledge and experience of using, adapting and creating algorithms is important both for the purposeful use of school algebra and arithmetic, and for efficient work with digital technologies in many contexts, such as mathematical modelling, technological design, and systems design. The National Curriculum should include invention, comparison and evaluation of the power and value of different methods, rather than only offer or expect 'given' methods.

35. The idea of using mathematics to model, and hence analyse, a range of real world problems should be given much more prominence than it has at present, and this should involve the analysis of real data using modern technology.

36. ACME has recently completed some relevant work on the essential arithmetical understanding that needs to develop at primary school in order for learners to be able to access the current secondary curriculum.

Key Stage versus year-by-year approaches

37. It is important to recognise that, although mathematics as a subject develops logically, with more advanced work relying on earlier ideas, students do not simply work through a list of topics in order. A mathematical idea needs to be encountered several times, in an increasingly sophisticated way, and in a variety of contexts, before it is fully grasped. Given this need for a spiralling rather than simply linear progression, setting out topics on a year-by-year basis is not likely to lead to the best overall outcome. It is also important to recognise that people differ in their rate and pattern of development – a Key Stage approach provides flexibility to cope with all parts of the spectrum.

38. The National Curriculum should therefore describe a realistic range of desirable outcomes for each Key Stage and core experiences (i.e. topics studied, tasks and activities undertaken, ideas exposed to) that all students should have in that Key Stage, together with the valued competencies and attitudes that are necessary for further confident progression. This would help students learn/achieve the desirable outcomes and would make the curriculum more coherent.

Levels of achievement

39. We recognise that teachers need to be able to assess students' progress formatively, and the need for accountability to parents and other stakeholders, but we believe that the current levels of achievement in the programme of study are unhelpful. In particular:

    - Levels as comparison between subjects are meaningless

    - Levels suggest that mathematics learning is linear rather than spiralling, and that students progress in equal intervals and at uniform rates across the curriculum and even between

mathematical topics. This is not consistent with our understanding of either the nature of mathematics or of mathematics learning.

- Teachers, especially if they are not mathematics specialist teachers, can be tempted to teach exactly what is required for the next test or next level instead of laying the foundations for understanding overarching mathematical ideas.

- Levels have drawn teachers' attention to what is possible to achieve but have also put restrictions on the assessment system

- An effect of attempting to equate a given level in different topics, say algebra and data handling, is that examination papers (eg GCSE) are uneven in difficulty and candidates concentrate on the easier topics, both in the examination and in their preparation for it. This leads to a distortion of teaching.

40. If Levels are to be removed, it is essential that alternative systems of assessment that support progress and allow accountability for parents and other stakeholders are developed in their place, based on measures of what we value rather than those things we can easily assess. The process of constructing an alternative framework is difficult and can be as influential as the Curriculum it supports – thought needs to be given to ensure that an effective system is in place.

International evidence

41. The Review group should refer to the following research founded by the Nuffield Foundation:

    Askew, M., Hodgen, J., Hossain, S. & Bretscher, N. (2010). *Values and variables: Mathematics education in high-performing countries.* London: Nuffield Foundation

    This reports aims to answer the following questions:

    - What is the range and type of research evidence from countries with high performance in mathematics that gives insights into the reasons for their relatively high position?

    - What constitutes high performance in mathematics learning and what factors appear to contribute most to achieving it?

    The following countries were studied in the project: Singapore, Hong Kong, Korea, China, Chinese Taipei (Taiwan), Japan, the Netherlands, Belgium, Hungary, Latvia, Czech Republic, Finland, Australia, Russian Federation, New Zealand, Slovenia and Slovak Republic.

42. ACME and SCORE (Science Community Representing Education) have also assembled a list of studies, reports and examples of curricula that should be consulted. This can be accessed via http://www.delicious.com/NationalCurriculumSTEM

How children learn

43. For information on how children learn the Review group should consult the following study from the Nuffield foundation: *Key understandings in Mathematics Learning:* http://www.nuffieldfoundation.org/sites/default/files/P1.pdf

44. The aim of this report is to offer a synthesis of research on mathematics learning by children from 5-16 and to identify the issues that are fundamental to understanding children's

mathematics learning. Three main questions regarding key understandings in mathematics are explored:

- What insights must students have in order to understand basic mathematical concepts?
- What are the sources of these insights and how does informal mathematics knowledge relate to school learning of mathematics?
- What understandings must students have in order to build new mathematical ideas using basic concepts?

Pages 8 and 9, in particular, offer a description of the core knowledge and ways of thinking that are needed for students to get started with mathematics.

45. The Review Panel should also refer to:

    - Kilpatrick, J., Swafford, J., and Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Research Council.

      This book provides a research-based view of how younger school students learn mathematics, and advice about best ways of teaching. Most importantly, it presents a fully justified view of mathematics as consisting of several strands of proficiency: procedural fluency, conceptual knowledge, strategic competence, adaptive reasoning and productive disposition. All these are needed in order for students to become competent with mathematics in later study and outside.

    - Swan, M. (2009) *Mathematics Matters: a final report*. London: NCETM

      This report presents the characteristics of effective teaching of mathematics in ways which ensure that all strands of mathematical proficiency are developed, through teaching and curriculum which include:

        - fluency in recalling facts and performing skills
        - conceptual understanding and interpretations for representations
        - strategies for investigation and problem solving
        - awareness of the nature and values of the educational system
        - appreciation of the power of mathematics in society

      The data is drawn from discussions with experienced teachers and teacher educators from across England. While some aspects are common to all teaching, and are subject to pedagogic decision-making, others are specifically mathematical in nature and contribute to an entitlement for students to develop mathematical reasoning as a central part of their knowledge.

    - Cockcroft, W.H. (1982) *Mathematics Counts*, London: HMSO

      The Cockcroft report was a major HMSO publication drawing on input from a wide range of stakeholders in mathematics education. Recommendations are made about content and nature of a mathematics curriculum for all. It takes the needs of industry and commerce into account as well as the requirements of HE. As well as making suggestions about curriculum, teaching and assessment it was also instrumental in the provision of local author mathematics advisory teachers to support professional change among teachers. It is impossible to summarise its many findings – it is a major source of wisdom in all aspects of mathematics education.

Implementation

46. The timescale for the implementation of the new National Curriculum will be challenging if there are to be significant changes. The difficulties of implementation are magnified by the decrease in support from Local Authorities and Higher Education Institutions, and by decreasing school budgets and by the demise of the National Strategies. It is notable that the two-year pay freezes for school teachers will expire in 2013, just as the new National Curriculum is due to be implemented.

47. Support for teachers will be very important in implementing a new National Curriculum; Continuing Professional Development (CPD) and good quality support material will need to be made available. The National Centre for Excellence in Teaching Mathematics (NCETM) will be a key source of advice in this area. Appropriate funding should be made available to support this CPD, including the production of guidelines and other necessary resources.

48. CPD that is focused on improving subject knowledge will be particularly important in the context of giving teachers more freedom to teach either for deep understanding or beyond the National Curriculum. If a new National Curriculum is to result in significant improvement, it requires focused development of non-specialist teachers of mathematics in particular – it cannot be seen in isolation. We have successful models of how this might be achieved in many current Mathematics Enhancement Courses and Mathematics Specialist Teacher programmes.

49. Material and support for teachers will need to build on what has been done before, by adapting current material where appropriate rather than trying to 'reinvent the wheel'; this would enhance teacher confidence in implementing any change.

50. Teachers will need an awareness of the full 5-16 National Curriculum to understand where what they are teaching fits in to the wider picture of progression.

51. Teachers will also need time and support to understand any new content and assess how current schemes of work should be adapted.

Advisory Committee on Mathematics Education

14 April 2011