Assessment in 14–19 Mathematics
Foreword

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ACME is an independent committee, acting as a single voice for the UK mathematical community, which seeks to improve the quality of mathematics education in English schools and colleges.

This report suggests that an effective way to address the problems raised by an overly burdensome examination system for 14–19 year olds studying mathematics, is for the Government and its agencies to give careful thought to the most appropriate types of assessment used, and to ensure that any trialling of innovative approaches is carefully and publicly analysed before they are rolled out nationally. I commend this report to policy makers, head teachers and those responsible for the teaching of mathematics throughout the education system.

ACME would be pleased to hear all views on this, its second self-initiated report, as well as having brought to its attention those other issues in mathematics education that are of particular concern. Whilst we cannot undertake to act on every request or piece of information we receive, we do pledge that everything sent to us will be read, considered fully, and taken into account.

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Assessment in 14–19 Mathematics

The Advisory Committee on Mathematics Education (ACME) seeks to offer advice to the Government and its agencies on all matters pertaining to mathematics education in English schools and colleges, including the recruitment and training of teachers of mathematics. It seeks to do this:

- by response to Government initiatives;
- by informal, and often confidential, discussions; and, from time to time,
- by the publication of reports on issues of special interest to the mathematical community.

This publication concerns such an issue – assessment of mathematics in schools and colleges. It has been clear for some time that the assessment regime has a major and increasing impact on the teaching and learning of mathematics, and that the mathematical community has strong views on this issue. Consequently, ACME has undertaken a consultative process on the assessment of 14–19 mathematics. Evidence was sought from the mathematical community and others (see page 12 for a list of contributors), and what follows are the fruits of this process. We start with a collection of principles upon which the text is built. The main text then highlights some specific areas of concern, leading to various recommendations. It should be noted that some, but by no means all, of the issues discussed here would apply to other subjects; however, they all appear here because of their strong effects within mathematics. Where we refer to colleges, this includes both sixth form and further education colleges.

1 Introduction

Assessment serves many purposes in education. For example:

- key stakeholders, including employers and universities, need authoritative information about the level of skills and knowledge of applicants;
- students need to know their level of achievement and to acquire qualifications;
- parents need to know how their children are progressing against national standards;
- assessment can guide and motivate learning for both students and teachers;
- the Government and its agencies use external assessment results to judge the performance of schools and colleges;
- national assessments provide some measure of standards over time.

In the discussion which follows, we distinguish between two distinct types of assessment, summative and formative.

- Summative assessment is defined here as the external, concluding assessment of a programme, for example, GCSE, or the Key Stage tests.
• Formative assessment is defined here as the internal assessments used throughout the teaching process that provide immediate diagnostic feedback to the teacher and student.

Although formative assessment is primarily the domain of the classroom teacher, we believe that there are some initiatives that might be taken nationally to support and encourage good practice. The distinctions between these two broad classifications are not always clear, and we discuss this briefly later.

2 Principles for assessment in mathematics

1. Assessment is important, but the purpose of education in mathematics is to learn mathematics, not simply to pass examinations.

2. Assessment should be compatible with good teaching and the effective learning of mathematics.

3. Assessment should be fit for purpose, and should, wherever possible and practicable, reflect the appropriate range of mathematical skills.

4. Summative assessment should not lead to harsher grading in mathematics than in other subjects.

5. Assessment should be manageable for teachers and for students.

6. External, summative assessment should be kept to a minimum.

7. Formative assessment is crucial to learning, and should be actively promoted.

8. Where computers and calculators are used quite properly in doing mathematics, this should be reflected in some measure in the assessment.

3 External, summative assessment

There is a clear need for external assessment at certain stages of development through the educational process. The key questions are: how much assessment is required; when should this occur; and what form should it take?

At present, students are assessed externally at the various Key Stages at ages 7, 11, 14 and 16 years (GCSE). Post-16 assessments at AS and again a year later at A2 complete the picture for many. There are also various vocational qualifications and the Key Skills assessment for others.

There are disadvantages in this proliferation of assessments. The present importance of external examination results to schools and colleges often means that teaching becomes unduly directed towards results, rather than towards students gaining a deeper and
Increasingly, evidence shows that excessive time is spent on revision, and that teaching, and textbooks, have become preoccupied with the needs of the assessment rather than fostering understanding and interest in the subject. There is considerable evidence of ‘teaching to the test’. Current funding arrangements in colleges can make this problem worse. A particularly damaging consequence of the proliferation of external assessment has been the production of league tables showing the ‘performance’ of schools, and these add to our concerns.

While the requirements of employers, universities and others certainly point to the need for external assessment at 18 years – and there are strong arguments for it at age 16 – we need to revisit the rationale for such extensive high-profile national testing at other times. The external assessments at the end of the Key Stages, while of some value to schools in providing a measure of pupil achievement, are used more frequently as measures of school performance. If such information is required, then it could be obtained by less intrusive methods, based perhaps on greater teacher assessment and judgement of pupils with sampling and moderation as appropriate. However, we should be wary of increasing internal assessment to the extent that it becomes a significant additional burden on teachers.

Where external summative assessment is appropriate, we need to consider how much assessment is required. The modular assessment structure associated with the introduction of Curriculum 2000 is a particular concern. Students take six module examinations in mathematics (and in all other A2 subjects), and may retake them as many times as they wish. The total volume and the frequency of assessment have become excessive, and, across the totality of their studies, students are quite simply confronted by too many external assessments. This is leading to an ‘examination mentality’, with a consequential negative impact on teaching and learning post-16.

The wider mathematical community believes that many of the negative effects of external assessment are serious, and that such assessment should be reduced to the minimum level to achieve its essential aims.

**Recommendation 1**
The Government should reduce the overall volume and frequency of external assessment in mathematics.

### 4 Goals of different 14–19 mathematics pathways

A clear principle of assessment is that it should be ‘fit for purpose’. This will almost certainly imply different types of assessment for different proposed pathways within a revised 14–19 structure.
At present there are two main forms of assessment used within 14–19 mathematics: timed, written examinations and formal coursework assessment. While other styles of assessment have been used – for example, pre-issue material for mathematical comprehension exercises – the majority of assessment falls into one of these two classifications. There is a strong consensus that the full range of mathematical skills is not being assessed appropriately and that this impacts on much classroom practice, with the consequence that important mathematical skills may be inadequately taught, or, worse, not taught at all.

The main areas of concern are:

a) the lack of a sound grasp of basic techniques, at all levels;
b) the failure to know, and to recall, important, basic facts;
c) the inability to apply mathematical skills to real situations;
d) the lack of confidence in modelling real situations with mathematics;
e) the inability to solve harder, multi-step problems.

One way to help improve facility in these areas would be to improve the design, and possibly the range, of the assessment instruments. Whatever approach is used, it is vital that the assessment style should be appropriate for each given pathway, and, most crucially, should be compatible with good teaching and learning in mathematics. This has not always been the case.

The introduction of coursework at GCSE level, for example, was expected to enhance the range of mathematical skills taught and assessed at all levels. It has largely failed to do this, and a predominantly negative attitude to coursework has developed over the years. The current assessment regime for the data handling coursework as a means of assessing Assessment Objective Four (A04) for students at GCSE, especially more able students, is a cause of particular concern. For some students, however, coursework can enhance their experience of the subject and be both motivating and challenging, and this is especially (but not exclusively) so for those who experience difficulty with the subject. This would suggest that the type of coursework, and the balance and weighting of this component, should vary across the different proposed pathways outlined within the Smith Report\(^1\). The Tomlinson proposal, where formal assessed coursework is no longer a requirement of each subject but becomes part of the overall assessment of the ‘diploma’, would fit well with this\(^2\).

A second illustration concerns the change to a modular structure of learning post-16 with the introduction of Curriculum 2000. While modular courses in mathematics existed before in mathematics (with a significant take-up

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of students), it was Curriculum 2000 that introduced a common modular structure in all A-level courses in mathematics, and, indeed, across all A-levels. Perhaps more significantly it introduced the concept of the AS level examination – a one-year staging post in the sixth form. While this is worthwhile for those who intend only a one-year course in mathematics, or settle on this during their studies, it has, as we will now explain, proved to be a bad idea for those taking, or intending to take, the full A-level in the subject.

While the modular assessments in the earlier mathematics courses provided useful feedback for teachers and students en route through the course, the present structure introduced high stakes formal assessment at the end of the first year, creating significant pressures for mathematics learning during this formative period of a student’s studies. Assessment too early in the programme does not allow students the time to consolidate and practise technique, nor come to any deeper understanding of concepts; yet these are crucial in mathematics. The poor results and high failure rates arising from the AS mathematics examinations deterred disproportionately many students from taking the subject on to the second year, where they may well have come to a greater understanding of the material and achieved more success. The piecemeal approach to the subject encouraged by much current modular assessment is not conducive to creating a coherent view of mathematics, nor to engendering excitement and interest. The nature of the questions set in the assessments has led to insufficient testing of higher-level skills, as well as of basic techniques; yet these are crucially important skills required for progression in mathematics.

Nevertheless, modular assessment can have benefits for many students, especially those requiring encouragement and reinforcement. A further positive feature of modular courses is that they require good syllabus coverage, and the nature of the assessment means that it is difficult to avoid significant chunks of the specification. Furthermore, it reduces what, for many students, is the significant stress of the single end-of-course examination. To avoid the potential dangers for mathematics, while exploiting some of the positive virtues for students, modular courses and their assessment need to be carefully designed, and be appropriate to the abilities of the students being assessed, and to the range of skills they seek to assess.

These two specific examples provide evidence of how an inflexible structure, and its associated assessment, can have a negative influence on the teaching and learning of mathematics for many students.
Recommendation 2
Assessment mechanisms in mathematics should reflect the goals of different 14–19 ‘pathways’ to a qualification. There should be no expectation that the mechanisms adopted for one pathway should be applied to other pathways, nor that mechanisms used in mathematics need coincide with those in other subjects. In particular, standardised modular structures and the use of coursework assessment should be used only as appropriate.

Recommendation 3
New assessment regimes in mathematics should always be trialled, and the results should be in the public domain. Before wider adoption, the results should be analysed carefully to ensure that the changes will achieve the desired aims.

5 Management of change in assessment regimes

Innovation and change (as well as periods of stability!) are a prerequisite for a healthy educational system, but the management of change is a delicate matter. When the Government or its agencies propose change, there is a natural wish to see this done as soon and as widely as possible. In practice, too speedy an implementation of change in education can be counterproductive. We believe that this is now more widely understood, and that it is recognised that there are substantial benefits to trialling proposed changes, with analysis, modification and improvement before a decision on general implementation. We urge that the intentions of any reforms are articulated, that trial results are placed in the public domain, and that the resulting analysis weighs the trial’s success in relation to its intentions. We also urge that the wide range of experience and expertise within the mathematical community is fully engaged in this process.

6 The Application of Number Key Skill and its replacement

The Application of Number (AoN) Key Skill is available at levels 1 to 4, outside the GCSE and A-Level structure. The qualification is divided into two equally weighted components: an external test and an internally assessed portfolio of work that is externally moderated. Students are able to claim exemptions from external tests by using ‘proxy’ qualifications. The whole process of proxy qualifications was highlighted in the Smith Report, and careful consideration needs to be paid to this. The AoN qualification has recently been adjusted and new standards have been in place from September 2004. However, even with these adjustments, there is still concern among teachers about the validity of the qualification.
The focus of AoN is primarily on numerical work, some basic statistics and topics from shape, space and measure. The range of skills assessed is limited, and the content is often seen by practitioners, and by students, as boring. Students often comment that the work they have to cover is a repeat of material that they were unable to understand at GCSE, and is not relevant to their other post-16 studies.

External tests for AoN levels 1 and 2 are multiple-choice. These are available either in a paper-based format or `on-line' on demand. The external test for level 3 is split into two sections: short-answer questions and extended-answer questions. Although the term `portfolio' is used to describe the internal assessment, students have to produce `overarching' pieces of work to demonstrate their ability to collect information, analyse it, present it and then interpret their findings. In this sense, the work becomes rather more like a piece of coursework.

It is important for students to develop Key Skills in mathematics. Although `generic' qualifications have advantages in that they are easy to administer on a large scale, it has proved difficult to integrate AoN into vocational areas, and the motivation of students who have a range of future goals is therefore an issue. In this context, it is perhaps sensible to consider Free Standing Mathematics Qualifications (FSMQs). Some of these are assessed through the use of a portfolio and external examination, and differ from AoN in that their specifications are designed so that students can demonstrate progression over time within their portfolio work. There is also a degree of flexibility in the content required which makes it easier to integrate material within vocational areas. Some use examination pre-release materials, allowing students to gain an understanding of a context before the examination. (A criticism of the AoN tests is the degree of literacy required to approach them successfully.) Overall, particularly at levels 1 and 2, the assessment mechanisms of the FSMQs appear to be better designed to match the skills to be assessed. Lessons should be learnt from both these qualifications when designing any future 14–19 programme related to AoN. Again, the Smith Report highlighted the effect that funding mechanisms can have on the selection of appropriate mathematics qualifications for students.

**Recommendation 4**
The revision of Application of Number assessment should be more compatible with the learning style and aspirations of the students it is assessing.

### 7 Rationalisation of 14–19 assessment systems

A feature of the English assessment regime, which differs from that of many other countries (and which also contrasts with the single provision of assessment of Key Skills and of Key
Stage tests), is the existence of three main Awarding Bodies. This multiplicity has the effect of creating competition between Awarding Bodies, one consequence of which is the stimulation, on occasion, of useful innovation.

However, there are negative consequences:

(a) It is visibly wasteful that significant resources are invested in the provision of appropriate specifications and examinations by Awarding Bodies, and, consequently, in the resulting comparisons of standards by the Qualifications and Assessment Authority (QCA). This is likely to become worse when we have a number of distinct pathways within the 14–19 structure.

(b) Schools and colleges are pressured to seek the options most likely to produce good results, with Awarding Bodies pressured to produce results which will satisfy their customers.

The need for continuing innovation is the strongest argument in favour of multiplicity of provision, but such provision does not necessarily either foster or encourage innovation. It does seem unnecessary, for example, to have all three main Awarding Bodies providing assessments of each module within the A-level programme. A particular disadvantage arising from this is that, for smaller entry subjects (such as Further Mathematics), candidates are spread thinly over the different modules offered by the Awarding Bodies and, consequently, there can be a reluctance among publishers to produce material to support the teaching of those modules where the entry is very small. We therefore recommend a reduction in the duplication of assessment, while ensuring that the system welcomes innovation. This does not imply having a single Awarding Body, but suggests, for example, that only one Awarding Body should be responsible for any particular pathway.

**Recommendation 5**

Where duplication exists in the assessment of 14–19 mathematics, the Government should rationalise assessment by the different Awarding Bodies.

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8 The use of computers and calculators in 14–19 mathematics learning and assessment

The appropriate use of computers and calculators can enhance and enrich the teaching and learning of mathematics. It is highly motivating for many students and continued and further use should be encouraged. Assessment mechanisms should recognise and reward the proper use and mastery of computer software. However, we must recognise that such assessment can be a problematic and contentious issue. For example, there is disagreement about the use of calculators in mathematics
examinations at GCSE and AS/A2 levels. The present arrangement is to allow calculators selectively, although some argue for wholesale use for all assessments and others argue for a reduced usage. What needs consideration is the extent to which the assessment measures the student’s effective usage of the calculator, and does not detract from the assessment of important mathematical processes and skills. Similarly, assessment of the use of spreadsheets and other software packages could be envisaged for appropriate work.

Computers have the potential to hold data banks of questions that could support teachers in diagnostic work or in assessment more generally. This could provide a convenient means for assessment-when-ready, where tests could be randomly constructed from a national databank, thereby supporting teachers in their assessments of students. However, mathematics is difficult to enter into a computer, and so the major type of computer-based assessment in mathematics tends to be short, structured questions which are generally multiple-choice. Even with the current limitations on computer software, on-line testing might well be appropriate for the assessment of some basic skills and techniques, but higher-level skills, such as being able to string together a coherent solution to a non-trivial problem, cannot (yet) be assessed in this way.

In the future, on-line testing could be effective and useful as a means of assessing some aspects of mathematics, and is likely to play a role as one of the tools available for both formative and summative assessment. We will need to explore more fully the current developments in these areas, including work at the Open University and in other Higher Education Institutions in this country and abroad.

At the same time, we must also be aware of the resource implications for both schools and colleges when entering into computer-based assessment. Although access to technology is improving, there is still a significant input needed to dedicate resources specifically for use within the mathematics classroom. Funding mechanisms must ensure that students in both schools and colleges are placed on an equal footing and have parity of access to appropriate resources.

**Recommendation 6**
The Government and its agencies should encourage more development work and research to examine ways in which the appropriate use of computers and calculators in 14–19 mathematics can be assessed.

**Recommendation 7**
More research is needed on the use of computer-based assessment in 14–19 mathematics.
Formative assessment is ultimately the responsibility of the classroom teacher, but its importance is such that we believe national initiatives may help encourage good practice in the classroom. Formative assessment may take many forms, including diagnosing students’ strengths and weaknesses from classroom activities such as focused questioning, marking homework and coursework, class tests and internal school examinations, and using these to give immediate feedback to students on how to improve their performance and to help plan the next stages of learning. We note, however, that there can be a conflict between the formative nature of this work and the use of it in a summative way – for example, when teacher assessment contributes to the assessment of external qualifications.

Overall, there is strong evidence to suggest that formative assessment is central to mathematics teaching and learning, and that it should be common practice among teachers of mathematics. However, OfSTED reports that this is not yet the case. There is thus a clear need for the methods and benefits of formative assessment in mathematics to be promoted in schools and in sixth form and further education colleges. Appropriate professional development programmes will play an important part in improving the expertise of mathematics teachers in the techniques of effective formative assessment in the classroom.

Recommendation 8
Formative assessment in 14–19 mathematics should be strongly promoted, particularly through the initial training and the professional development of teachers.

The QCA provides some support and guidance materials on ‘Assessment for Learning’ in mathematics. It appears that the existence of these materials is not widely known, nor are they widely used, in the 14–19 sector. Those that commented on them in our research found them very valuable in pointing teachers toward good practice in assessment and recording. Other material, including research material, is available, and would be useful if more widely disseminated to teachers. There is also a real need for materials that help integrate formative assessment into the teaching and learning of mathematics. Existing and future materials should be promoted to both schools and colleges.

Recommendation 9
High-quality support material on formative assessment in 14–19 mathematics should be made readily available and widely promoted.
10 Recommendations (summarised)

1. The Government should reduce the overall volume and frequency of external assessment in mathematics.

2. Assessment mechanisms in mathematics should reflect the goals of different 14–19 ‘pathways’ to a qualification. There should be no expectation that the mechanisms adopted for one pathway should be applied to other pathways, nor that mechanisms used in mathematics need coincide with those in other subjects. In particular, standardised modular structures and the use of coursework assessment should be used only as appropriate.

3. New assessment regimes in mathematics should always be trialled, and the results should be in the public domain. Before wider adoption, they should be analysed carefully to ensure that the changes will achieve the desired aims.

4. The revision of Application of Number assessment should be more compatible with the learning style and aspirations of the students it is assessing.

5. Where duplication exists in the assessment of 14–19 mathematics, the Government should rationalise assessment by the different Awarding Bodies.

6. The Government and its agencies should encourage more development work and research to examine ways in which the appropriate use of computers and calculators in 14–19 mathematics can be assessed.

7. More research is needed on the use of computer-based assessment in 14–19 mathematics.

8. Formative assessment in 14–19 mathematics should be strongly promoted, particularly through the initial training and the professional development of teachers of mathematics.

9. High-quality support material on formative assessment in 14–19 mathematics should be readily available and widely promoted.
List of contributors

The Committee held evidence sessions with:
- Assessment and Qualifications Alliance (Martin Taylor, Research Officer and Anne Trant, Senior Subject Officer GCSE)
- King’s College London (Professors Alison Wolf and Margaret Brown)
- Mathematics in Education and Industry (Roger Porkess)
- OfSTED (Lynn Churchman HMI, Specialist Mathematics Consultant, and Alex Falconer HMI, post16)
- QCA (Alice Onion, Principal Adviser, Mathematics)

Communications were received from:
- Association of Teachers of Mathematics
- Association of Mathematics Education Teachers
- Alan Bloomfield
- British Society for Research into Learning Mathematics
- Conference of Heads of Department of Mathematical Sciences
- Professor Neville Davies
- Caroline Dawes
- Rosemary Emanuel
- Dr Tony Gardiner
- J Gibson
- Jenifer Golding
- Peter Holmes
- Institute of Mathematics and its Applications
- London Mathematical Society
- Mathematical Association
- National Association of Mathematical Advisers
- National Association for Numeracy and Mathematics in Colleges
- Royal Statistical Society
- Scottish Mathematical Council
- Peter Thomas
- Owen Toller
ACME and Royal Society/JMC mathematics education reports

Continuing Professional Development for teachers of mathematics
(15 page report of ACME’s first self-initiated project, December 2002, 2 page summary also available)*

Teaching and learning geometry pre-19
(88 page report of a Royal Society/JMC working group, July 2001, 3 page summary also available)**

Mathematics education pre-19
(4 page statement by the Royal Society, May 1998)

Teaching and learning algebra pre-19
(72 page report of a Royal Society/JMC working group, July 1997, 4 page summary also available)**

* Full text of this report can be found on ACME’s webpages at www.acme-uk.org
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Copies of these publications can be obtained by sending a self-addressed and stamped envelope to:
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