There is a wide consensus amongst politicians, employers, higher education tutors and others that students in England should, like their counterparts in the vast majority of other countries, continue to study mathematics to the age of 18. ACME has spent the past year considering how this can be achieved for those students ready to progress beyond GCSE Mathematics.

The reason for this consensus is that mathematics is becoming increasingly important in both employment and in higher education. The country needs more people with skills in mathematics beyond those required to obtain at least a C grade in GCSE Mathematics. The current deficiency is highlighted in the recent ACME Report, *Mathematical Needs: Mathematics in the workplace and in Higher Education*. The essential question to address is how to overcome this shortfall.

In England, the number of students studying A level Mathematics has been rising steadily for some time. Admittedly, this rise comes after a serious drop in participation following the introduction of Curriculum 2000, but nevertheless increasing numbers of students are opting to study AS and A level Mathematics. Numbers studying AS and A level Further Mathematics have also risen dramatically. However, despite this positive news, we estimate that each year at least 250,000 students in England with a C or above in GCSE Mathematics choose not to study any mathematics after the age of 16.

Increasing the number of students studying mathematics beyond GCSE by anything like 250,000 requires a coherent strategy and many years of investment. Providing insight into the way this can be achieved has been ACME's goal.

We are convinced that dramatically increasing participation in mathematics for these students will not happen without some form of accountability 'stick'. ACME is convinced that a number of approaches will be required to change behaviours, for example funding requirements, accountability measures and a clear signalling of higher education and employment requirements.

Today, ACME is publishing two papers. The first, *A strategy for improving provision and participation* provides an overview of the fundamental changes needed to the structure and type of mathematics qualifications. The second paper, *Planning for Success*, describes the steps that will need to be taken to make this work. We are setting a demanding challenge. Many agencies and organisations will need to work together, alongside government, so that the complex ecosystem that drives students, schools and colleges ensures 21st Century students are equipped with the mathematical capabilities that they are going to need in their personal and working lives.

ACME will continue to take a keen interest in this area, but now is the time for Government, agencies, universities, schools and colleges to take these proposals forward. This is not a simple job – it will take time, investment and careful expert management to achieve success.

*Stephen Sparks*
Chair, Advisory Committee on Mathematics Education
In response to Michael Gove’s speech at the Royal Society in June 2011, and to recommendations in reports such as Mathematical Needs\(^1\) and Is the UK an Outlier?\(^2\), ACME has set out a strategy for dramatically increasing participation in post-16 mathematics\(^3\).

ACME has identified that the majority of students with a grade C or above at GCSE are not well served by the current qualifications provided in schools and colleges. In this paper, we set out the requirements and rationale for the development of a new post-16 qualification to sit within a clear, simple suite of distinctive and well-understood post-GCSE qualifications. In an accompanying paper, Planning for Success, we describe the actions that will need to be taken to improve participation through this new qualification, including addressing teaching capacity, stakeholder buy-in and funding levers.

This new qualification should have an emphasis on mathematical problem solving and should support students on academic, mixed and vocational programmes. There will need to be incentives for schools and colleges to provide an appropriate suite of qualifications to meet the needs of all their students. There will also need to be incentives for students to take these qualifications. The qualifications should form part of a structured study programme and accountability measures should support this.

It is essential that the key goal for any new post-16 qualifications in mathematics should be to develop and consolidate understanding of mathematics through motivation and engagement. This is more likely to be achieved if the qualifications concentrate on using mathematics to develop and solve problems in realistic contexts.

ACME recommends that:

- There should be a concerted drive to dramatically increase the participation rate of those studying mathematics beyond GCSE level.
- The primary incentive to study mathematics should come from its inclusion in recognised and structured programmes of study, clearly understood, advocated and valued by higher education and employers.
- AS Mathematics should be retained, and promoted, both as a qualification in its own right and as the first half of the full A level Mathematics within the post-GCSE qualification suite. It should be clearly seen as both the sole route into advanced mathematics and the essential minimal mathematics qualification of choice for future scientists.
- A new qualification should be developed and introduced as part of wider A level reforms. This qualification should:
  - Be distinct from A level Mathematics, with an emphasis on solving realistic problems, using a variety of mathematical approaches, and should be for students not currently doing AS or A level Mathematics
  - Give students the confidence to consolidate their understanding of mathematics by using and applying mathematics already learned in GCSE and new mathematics beyond GCSE developed during the course.
  - Have a smaller volume than AS level and be designed to be studied over two years.
- The mathematical content of other academic and vocational qualifications should be raised to realistically suggest how mathematics is used in these contexts and to mutually reinforce mathematical understanding through the simultaneous study of the new qualification.
ACME’s Mathematical Needs\(^4\) reports described how the mathematical demands of almost all university courses and workplaces are increasing. ACME has estimated that around 330,000 young people each year need some post-GCSE experience of mathematics. Yet the Nuffield Foundation report *Is the UK an Outlier?\(^5\)* found that the level of post-16 participation in mathematics was extremely low in England compared with other countries.

At the moment, post-16 participation in mathematics is highly predicated on prior achievement. The overwhelming majority of those taking A level Mathematics are those with grade A or A*, but even at this level participation is low – less than 50% of students with a grade A go on to study AS level Mathematics. By comparison, around 1% of those with grade C in GCSE Mathematics continue to AS Mathematics.

*Table 1 – Progression from GCSE Mathematics to AS/A Level Mathematics*

<table>
<thead>
<tr>
<th>Grade</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A*</td>
<td>5%</td>
<td>11%</td>
<td>17%</td>
</tr>
<tr>
<td>Progression rate from GCSE to AS Mathematics by GCSE Mathematics grade(^7)</td>
<td>79%</td>
<td>48%</td>
<td>15%</td>
</tr>
<tr>
<td>Progression rate from GCSE to A level Mathematics(^8)</td>
<td>73%</td>
<td>34%</td>
<td>6%</td>
</tr>
</tbody>
</table>

We estimate that each year there are at least 250,000 students who achieve a grade C or better at GCSE but who do not progress further with the subject. Many of these students will become primary school teachers, Health Service professionals, and workers in a wide range of public and private sector occupations. They, and many others, would benefit enormously from a sustained, deep and confident understanding of mathematics beyond GCSE\(^9\). Under the existing system, most of these students will have forgotten much, or all, of their mathematics by the time they are required to use it.

The utilitarian argument for increasing post-16 participation in mathematics is strong, but the case is bolstered further by considering mathematical study as a cultural activity\(^10\) – as an end in itself, to which a much larger proportion of the population should be exposed as a matter of course in a developed society.

ACME has therefore set itself the task of recommending measures that will increase take-up of post-16 mathematics for students moving beyond GCSE. There is one recent significant success that can be built on; the recent increases in numbers studying AS and A level Mathematics (and Further Mathematics), albeit following a plummet in the early part of the century. The numbers taking A level Mathematics continue to increase year on year, and have risen by 62% since 2004\(^11\). This shows that students will take, and schools and colleges will offer, post-GCSE mathematics courses. The challenge is how to extend the provision to cover the full range of students.

There is a range of qualifications available beyond GCSE: AS and A levels in Mathematics and Further Mathematics, Free Standing Mathematics Qualifications (FSMQs) and AS Use of Mathematics. However, aside from AS and A level Mathematics and Further Mathematics, the other qualifications currently have relatively limited take-up. The largest is the Additional Mathematics FSMQ offered by OCR (16,730 entries in 2012\(^12\)).

In this paper we consider the possible ways forward and describe the range of provision that should be in place for all students who have already obtained at least a grade C in GCSE Mathematics\(^13\). In particular, we outline the aims for a new two-year post-16 mathematics qualification of similar level to AS Mathematics, but smaller in size and with a distinctly different rationale and focus. We explain why we think it is the appropriate solution, and how the qualifications fit together. In *Planning for Success* we then describe all the measures that need to be in place to support the qualification structure.
Ensuring that there are appropriate qualifications for all students is just one step towards increasing provision. Consideration also needs to be given to whether these courses are compulsory or voluntary, and whether to achieve this through accountability measures and/or funding requirements. The decision about whether and how to take this step will have implications for the design of any new qualification.

Most countries with a high level of participation in mathematics post-16 have a variety of qualifications or courses available. Forthcoming research from the Nuffield Foundation will describe the different content and structure of mathematics provision in a number of countries. Countries with high participation rates recognise that students will have differing needs, depending on their intended next steps and prior achievement, but, where compulsory, mathematics courses sit alongside other compulsory subjects, such as the first language. ACME is therefore not in favour of introducing compulsion solely for mathematics; it should be done as part of a package in which some other subjects also have an element of compulsion.

ACME remains convinced that in the long term the most effective way to increase post-16 participation in mathematics would be to introduce some form of baccalaureate. This would not require wholesale revision of the constituent courses. A structured programme, which includes some mathematics, could be created from existing courses. If sufficiently valued by higher education and employers, such a baccalaureate would solve the dilemma of incentivisation versus compulsion.

School and college funding requirements can also be used to make study of post-16 mathematics effectively compulsory without the need for a baccalaureate model. ACME is also concerned that having mathematics as the only compulsory subject to be studied post-16 study could be counterproductive particularly regarding student motivation. In this situation, mathematics courses would need to be well designed to motivate students. Moreover, compulsion will not guarantee that learners value the qualifications. Similarly problematic are qualifications that are valued by students but not by stakeholders (e.g. HEIs) and as a result struggle to gain currency (e.g. Use of Mathematics). The qualifications included in funding requirements would have to demonstrate their worth both to the students and end users.

In our proposals, we describe a qualification suite that meets the needs of today’s students and also builds on the current qualification structure. The new qualification we propose would sit neatly alongside three A levels, a Level 3 BTEC or a vocational programme; in the future, it could become the minimal mathematics component of an Advanced Baccalaureate or a Technical Baccalaureate, or a structure that encompasses both academic and vocational education.

Recommendation 1:
ACME recommends that the incentive to study mathematics beyond GCSE should come from its inclusion in structured programmes of study, clearly understood, advocated and valued by higher education and employers.
In the remainder of this paper, ACME focuses on the mathematics qualifications suite. In particular, we consider the qualifications needed to develop the mathematical knowledge, skills and confidence of students moving beyond GCSE and how these qualifications should relate to each other.

As a result of our consultation and evidence gathering we have identified a governing principle, henceforth referred to as **parsimony**, according to which we should avoid the proliferation of new courses. First, higher education and employers are more likely to recognize and value a smaller number of qualifications. Secondly, schools and colleges vary in size and resource. For example, a third of centres offering A level Mathematics have 10 or fewer entries; 60% of centres have fewer than 100 students in total at the end of KS5 each year, and together they teach around a quarter of all KS5 students. Most institutions are unlikely to be able to offer more than one additional qualification.

There should be a recognisable qualifications structure encompassing mathematics provision beyond GCSE. This structure should contain a limited number of strands, each with a different focus. This structure should seek to ensure that achievement beyond GCSE can be readily recognised by all. Moreover, ACME considers that participation in mathematics post-16 should be continuous until (at least) age 18.

In summary, ACME proposes a national qualifications structure beyond GCSE Mathematics as follows:

- A new two-year qualification to meet the needs of students not currently served well by existing provision.
- AS and A level Mathematics and Further Mathematics should be retained in order to meet the needs of students likely to progress to mathematically demanding further study or employment.

In addition, relevant mathematics should be embedded in academic and vocational qualifications and students should be supported as they move beyond GCSE so that both routes described above are open to them.

### 3.1. The role of AS Mathematics

All the A level mathematics courses fare well in Ofqual’s recent international comparison of qualifications and by far the most popular post-16 mathematics qualification is AS Mathematics. This course provides a coherent minimal introduction to calculus and ACME considers that increasing participation in AS Mathematics is an essential component in improving the preparedness of students for further study and employment. An increase in numbers taking AS Mathematics can also lead to subsequent increases in students taking A level mathematics. Numbers taking AS Mathematics have doubled in the last decade, and it is essential that this trend continues.

It follows that we must do everything possible to optimize AS in its own right and to support the present increase in its numbers. This may require small changes to AS itself, but primarily it requires that we understand and remove the barriers to take-up of AS. These barriers are essentially of three types: lack of motivation, restrictive entry requirements and lack of knowledge and skills.

#### 1. Motivation

There are a number of factors influencing students’ motivation to study AS Mathematics. At 16, many students are very unclear about their post 18 intentions, whilst others may have a lack of understanding as to how AS Mathematics would support their current or future studies. Many will find that their level 3 study itself results in a change of direction, ambition and intention. Although some young people under- or over-estimate their potential for academic success, others have the prerequisite skills and knowledge to study AS Mathematics, but make a positive decision to study something other than mathematics. Additionally, some students will have had mediocre experiences of mathematics pre-16, making them unlikely to opt for AS Mathematics (or any mathematics) for its own sake.
2. **Restrictive entry requirements.** AS Mathematics is a high risk subject choice and students, and schools and colleges know that students are more likely to achieve higher grades in other A level subjects. Very few students with grades C or B in GCSE Mathematics are admitted to mathematics at A level. Institutions are also concerned to protect their success rates. For all these reasons, schools and colleges are reluctant to offer such students places on an AS course.

3. **Knowledge.** There is a mismatch between the knowledge and skills of some young people with Grades C and B at GCSE and the demands of AS Mathematics. There are students who may not yet have the prerequisite skills and knowledge for AS Mathematics, but might either want to continue studying mathematics for its own sake, or be taking other courses for which continuing with mathematics beyond 16 would be beneficial (e.g. Sciences, Business Studies, ICT). These students should have the opportunity to study AS Mathematics.

It is clear that Mathematics is unusual among A levels in that teachers and students alike are very aware that success is unlikely without at least a grade A at GCSE. Addressing the entry requirements and knowledge barriers could be addressed by improving the progression between GCSE Mathematics and AS Mathematics and this might be considered as part of GCSE and A level reform. However, ACME believes that reducing the demand or content of AS to improve access would impact on its role in preparing students for entry to higher education. We do not believe that this could be considered as part of GCSE and A level reform. Hence it is ACME's view that any alternative qualification with common curriculum principles and structures.

ACME is also convinced that a clear distinction between an alternative qualification and AS Mathematics needs to be maintained. The need to maintain the distinction between the new qualification and AS/A level Mathematics has affected ACME's recommendations about the new qualification. Hence it is ACME's view that any alternative qualification should be smaller in size than AS Mathematics, should be studied over two years and should focus on realistic problem solving. It is essential that these new courses support students on both academic and vocational (and mixed) programmes.

ACME has considered a number of ways of achieving this, and also how best to ensure such a qualification can be taken alongside, rather than supplant, students' other qualification choices. It is also essential that these new courses support students on both academic and vocational (and mixed) programmes.

We are also guided by the parsimony principle and higher education, employers, schools and colleges would value provision of a recognisable qualification with common curriculum principles and structures.

ACME has been particularly concerned with the mathematical needs of students currently not studying mathematics beyond GCSE. ACME believes that these can best be met by introducing a new qualification with a much greater emphasis on realistic problems, using real data and IT, than is achieved in either AS Mathematics or GCSE Mathematics. ACME believes that these can best be met by introducing a new qualification with a much greater emphasis on realistic problems, using real data and IT, than is achieved in either AS Mathematics or GCSE Mathematics. ACME's **Mathematical Needs** research has shown that many jobs and HE courses, often outside the traditionally numerate disciplines, are becoming more quantitative in nature. This makes it increasingly important that young people become confident in selecting, using and applying quantitative methods, though they may not want to sacrifice one of their existing subject choices in order to do so.

ACME's **Mathematical Needs** research has shown that many jobs and HE courses, often outside the traditionally numerate disciplines, are becoming more quantitative in nature. This makes it increasingly important that young people become confident in selecting, using and applying quantitative methods, though they may not want to sacrifice one of their existing subject choices in order to do so.

ACME has been particularly concerned with the mathematical needs of students currently not studying mathematics beyond GCSE. ACME believes that these can best be met by introducing a new qualification with a much greater emphasis on realistic problems, using real data and IT, than is achieved in either AS Mathematics or GCSE Mathematics. ACME has considered a number of ways of achieving this, and also how best to ensure such a qualification can be taken alongside, rather than supplant, students’ other qualification choices. It is also essential that these new courses support students on both academic and vocational (and mixed) programmes.

Drawing this together, ACME sees desirable features of a new post-16 mathematics qualification (for those with a grade C or above in GCSE Mathematics) as follows:

- The qualification would need to engage students, by stressing relevance through the use of realistic contexts with realistic data, and via additional topics such as further statistics and probability approached through problem solving
- The qualification would be designed to give students a thorough grounding in mathematics by the process of consolidation and gaining of confidence by applying aspects of GCSE Mathematics, and via additional topics such as further statistics and probability approached through problem solving
- The qualification would be driven by realistic problems, motivated in students’ other interests, and should build on the positive work done in developing materials for Use of Mathematics and FSMQs.
- In order that it can sit alongside a current typical post-16 programme of qualifications, and be distinct from AS Mathematics and not in competition with it, our proposal is that a new two year qualification should be ‘long and thin’. Such a low-volume course is more easily resourced and timetabled, and should prove itself of disproportionate value by enabling skills, confidence and engagement with mathematics to advance rather than atrophy during the years between GCSE and higher education or employment.

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**Recommendation 2:**

AS Mathematics should be retained, and promoted, both as a qualification in its own right and as the first half of the full A level Mathematics within the post-GCSE qualification suite. It should be clearly seen as both the sole route into advanced mathematics and the essential minimal mathematics qualification of choice for HE science programmes.
• The assessment should be creative: extended case studies, open-book or advance-material comprehension papers and a range of other options should all be open to the awarding bodies.

• Some students who were unable to access AS Mathematics at 16 will find that success on the first year of this new course gives them the skills, knowledge and enthusiasm to enrol onto AS Mathematics. To enable this progression the first year of the course would need to be an end point in its own right.

• Awarding organisations might also wish to consider whether the second year be admissible as an AS Mathematics applied module, so that students need study only the new course and the pure core (C1C2) in order to achieve AS Maths in their second year.

There are risks to consider if a qualification two-thirds of an AS in size is introduced. AS levels have credibility with Higher Education and employers, and it might be harder for a smaller qualification with a new qualification title to gain this credibility. However, these risks need to be balanced against the flexibility of this proposal, including how such a qualification would sit alongside students’ other qualifications and the resources required to teach it. ACME is also concerned that introducing the course as an AS would risk the course being taught in one year and the subsequent introduction of an A level, neither of which would be desirable. On balance, ACME is convinced that any alternative mathematics qualification should be less than an AS in size.

ACME believes Awarding Organisations should be asked to explore the development of such a qualification, alongside undertaking further research with schools, colleges, higher education and employers about the potential credibility of a qualification of this size and shape.

Recommendation 3:
A new mathematical qualification, based on problem solving in realistic contexts, should be developed and introduced as part of wider A level reforms.
This qualification should:
• Be distinct from A level mathematics, with an emphasis on solving realistic problems, using a variety of mathematical approaches.
• Give students confidence in using and applying pre-16 mathematics.
• Have a smaller volume than AS and be designed to be studied over two years.
• After one year’s successful study, students should be eligible to enrol on AS Mathematics, having gained the interim qualification.
• The new qualification should be designed so that it is not in competition with either AS or A level Mathematics.

3.3. Existing qualifications
It is natural to ask whether current qualifications such as the AS in Use of Mathematics, the Free Standing Mathematics Qualifications (FSMQs) or the IB Mathematical Studies Advanced Level would meet the needs of the alternative provision outlined in this paper. We believe that they would not do so at present due to lack of parsimony, continuous study and currency.

• Parsimony: There are at present several FSMQs, as well as AS Use of Mathematics. The titles, size, aims, principles and coherence of these qualifications would need to be reviewed in order to address ACME’s concerns about parsimony.

• Continuous study: As it stands, AS Use of Mathematics is generally taught as a one year course and displaces one AS level choice. FSMQs are 60 GLH and usually taught in one year.

• Currency: We are also concerned that both AS Use of Mathematics and the FSMQs have not gained sufficient currency with higher education and employers. We believe that a fresh start, with a new approach to syllabus development and high quality assessment design in which higher education is fully engaged, is essential.

This is not to say that these existing courses could not be maintained, but we believe that a common size and shape of course, alongside a common name, would considerably strengthen the currency and hence take-up of such courses.

3.4. Embedding mathematics within other courses
ACME believes that to have a well-defined mathematics qualification suite, with a few, unified, generic mathematics qualifications, will facilitate greater use of mathematics in students’ other studies. We note that SCORE and Nuffield have recently published reports which highlight the lack of appropriate mathematics currently embedded within science and social science A level qualifications25,26, and recommend steps that should be taken to improve that situation. The reports (and others27) also begin to identify the supporting mathematics needed for these subjects pre- and during A level, and it is striking how much of this is in common. Awarding organisations should use this research and work with SCORE and Nuffield in order to consider appropriate mathematical content of any new qualifications and also any changes to AS and A level Mathematics.

Embedding mathematics in other courses is an important element of the strategy to increase participation. However, it should not be seen as an alternative to the other steps described in this paper, and should not count towards any targets set for increasing mathematics participation post-16 – the two must work together28. Embedding alone is unlikely to sufficiently improve students’ skills and understanding when they enter higher education or employment, in part because such skills would have been developed and used only in a particular context and for particular purposes. ACME wants nearly all students with a good pass in GCSE Mathematics to study for a further qualification in mathematics. To have students studying for this new qualification together with studying subjects that have more mathematics embedded within them can only lead to reinforcement and consolidation of their mathematical knowledge, skills and understanding.

In this way, the nation will develop more competent young people with strong and transferable mathematical skills.

Recommendation 4:
The mathematical content of other academic and vocational qualifications should be raised to realistically suggest how mathematics is used in these contexts and to mutually reinforce mathematical understanding through the simultaneous study of the new qualification.
Throughout this document, we use the word ‘mathematics’ to encompass statistics, and refer specifically to statistics where necessary.

DfE research report RR195
Subject progression from GCSE to AS Level and continuation to A Level

Figures taken from Table B; note that these proportions are based on the number of entries (731,900 for mathematics for the 2007/08 cohort) rather than the number of students attempting (609,700). The former includes attempts by these pupils in previous years, but is relevant in assessing the progression of this cohort.

Ibid, Table 1.1 – based on the progression of the 2007/08 cohort in the following two years

Ibid, Table 3.1

See ACME’s workplace case studies in Mathematical Needs: Mathematics in the workplace and in Higher Education


JCQ data (UK figures, all ages) available from www.jcq.org.uk: 52,788 entries in 2004; 85,714 entries in 2012. A level Further Mathematics has grown by 131% in this period.

Data provided by OCR.

It should be noted that there is also a need to consider the mathematics qualifications available to those who have not achieved at A*-C in GCSE Mathematics – these students represent a substantial proportion of the cohort. However, this issue demands separate attention, and is not the focus of this paper.

To be published in January 2013.

See discussion in a recent report by Pearson
http://thepearsonthinktank.com/2012/rational-numbers-investigating-compulsion-for-mathematics-study-to-18/


Calculated from information contained within the supplementary tables of GCE/Applied GCE A/AS and Equivalent Examination Results in England, 2011/12 (Provisional). There are a small number of extreme outliers for both of these figures – large providers of A level mathematics, and large providers of post-16 education.


See also ACME’s work on Developing Able Young Mathematicians

http://www.acme-uk.org/the-work-of-acme/proactive-projects/mathematical-needs-project


See footnote 21, and the discussion on Tim Gowers’ blog at http://gowers.wordpress.com/2012/06/08/how-should-mathematics-be-taught-to-non-mathematicians/

For example, it could be composed of 120 guided learning hours (glh) in total, 60glh – equivalent to one third of an AS level – in each of two years of study.

http://www.transmaths.org/

http://www.score-education.org/media/10033/score%20maths%20in%20science%20summary%20report.pdf

http://www.nuffieldfoundation.org/sites/default/files/files/Maths_in_A_level_Assessments_Nuffield_Foundation_WEB.pdf

http://www.bioscience.heacademy.ac.uk/ftp/reports/biomaths_landscape.pdf