ACME Advisory Committee on Mathematics Education

Raising the bar: developing able young mathematicians

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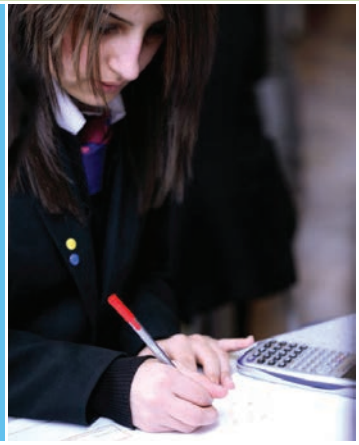
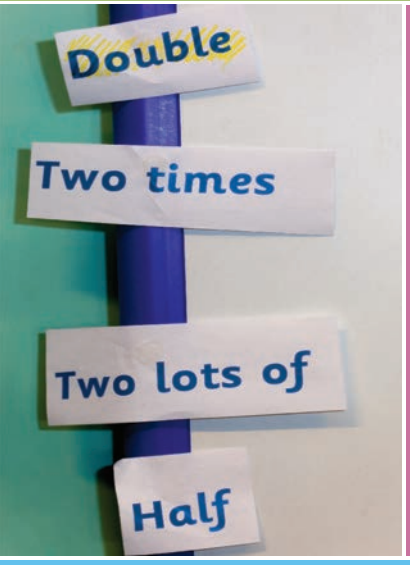


About ACME

The Advisory Committee on Mathematics Education (ACME) is an independent committee, based at the Royal Society and operating under its auspices, that aims to influence Government strategy and policies with a view to improving the outcomes of mathematics teaching and learning in England and so secure a mathematically enabled population. ACME acts as a single voice for the mathematics community.

This paper has been informed by a workshop on provision for able young mathematicians at the ACME conference, 10 July 2012, by a range of evidence relating to the effective development of young mathematicians, by a workshop with relevant experts organised by the London Mathematical Society, and by input from ACME's Outer Circle. Particular thanks are due to Tony Gardiner for his advice and support in the drafting of the paper.



Summary

At present, England is significantly underachieving in terms of developing able mathematicians, and this situation is now critical. It is necessary to increase systematically the number of young mathematicians with a robust and deep grasp of the range of mathematical ways of thinking and working. Students should have an engaging and challenging experience of school mathematics that will encourage many more to pursue mathematically-intensive courses at university.

Able mathematicians are defined in this paper as those students aged 5-16 who have the potential to successfully study mathematics at A level or equivalent. These students benefit most from an approach that aims to enrich and deepen the content of the curriculum, focusing on enabling students to achieve a deep mastery of the material. Acceleration through the curriculum promotes superficiality, not the true depth and rigour of knowledge that is a foundation for higher mathematics. The following principles should be adopted in order to meet the challenges outlined above:

- Potential heavy users of mathematics should experience a deep, rich, rigorous and challenging mathematics education, rather than being accelerated through the school curriculum.
- Accountability measures must be developed so as to fully align with these principles.
- Investment in a substantial fraction of 5-16 year olds with the potential to excel in mathematics, rather than focussing attention on the top 1% (or so), is needed to significantly increase national capacity in higher mathematics.

The Government should continue its investment in mathematics education by funding the development of an extended interpretation of the new National Curriculum, together with appropriate workforce development and resources to supplement those already available.

Background

Recent research-informed reports are unanimous in their conclusions that the UK needs *more* young people to study more mathematics more effectively, including at the highest levels[1,2]. As a nation we are constrained by a significant shortage of young people who are willing and able to pursue mathematically demanding degree courses and to enter mathematically demanding occupations. At school level, our performance in international measures of mathematics performance is relatively poor among developed nations[3,4]. There is also evidence that of those who perform at the highest level in international studies such as PISA, almost none in England come from state schools[5]. Poor performance such as this clearly affects our capacity to compete with other nations. In addition, there is evidence that even students who achieve good grades in mathematics are not necessarily confident to use the mathematics they have learned[1].

In short, the UK is not realising its potential in terms of growing and supporting able mathematicians. This is a huge waste that the UK can ill-afford, economically or socially, and it shortchanges individual young people: the problem requires urgent attention.

There have been a number of attempts to improve the education of England's able young mathematicians. However these schemes have not been as successful as anticipated[4], with a resultant cumulative loss of capacity, both for individuals and for the nation as a whole.

The government is currently in the process of implementing major changes in education, such as the review of the National Curriculum and changes to Key Stage 4 qualifications. It would therefore be timely to seize this opportunity to ensure that those changes follow the principles outlined in this paper to support young able mathematicians in the best way possible. Able mathematicians are defined in this paper as those students aged 5-16 who have the potential to successfully study mathematics at A level or equivalent.

Aims of this document

ACME's aims are to ensure that policy seeks to develop and support young able mathematicians 5-16 in order to:

- Systematically increase the number of young mathematicians with a robust and deep grasp of the range of mathematical ways of thinking and working.
- Provide these students with an experience of school mathematics that encourages many more to pursue mathematical studies at university.

Mathematics education takes place in three arenas: (i) in school, in class; (ii) in school, out of class; and (iii) beyond school.[6] This paper focuses on the first, and discusses the curriculum provision that should be made available and the policy measures needed to ensure that able young mathematicians receive quality first teaching[7]. In particular, the paper sets out how to improve provision for 5-16 students through an extended interpretation of the core curriculum, accountability measures, provision of substantial CPD for teachers, and the provision of resources (including textbooks).

Taken together, implementing the measures identified in this paper would begin to address the considerable mathematical shortfall identified in ACME's *Mathematical Needs* report[8]. Over time, these changes should become embedded as an integral part of the classroom experience, as indeed they already are in the best classrooms[9].

Principles

ACME's recommendations to support young able mathematicians are based on three key principles:

- **Potential heavy users of mathematics should experience a deep, rich, rigorous and challenging mathematics education, rather than being accelerated through the school curriculum.**
- **Accountability measures should allow, support and reward an approach focused on depth of learning, rather than rewarding early progression to the next Key Stage.**
- **Investment in a substantial fraction of 5-16 year olds with the potential to excel in mathematics, rather than focussing attention on the top 1% (or so), is needed to increase the number of 16+ students choosing to study mathematics-based subjects or careers.**

Principle 1) Experiencing a mathematics curriculum with depth and connections rather than acceleration

It is vital that those young people with the potential to excel in mathematics are well-supported in our schools. Young people in any classroom will have a range of mathematical needs, and at times this will necessitate particular provision for smaller groups of students within one class.

It is not unusual for those groups or individuals identified as able mathematicians to be allowed or encouraged to progress through the curriculum at a faster pace. Such acceleration in mathematics is often counterproductive. Acceleration encourages only a shallow mastery of the subject, and so promotes procedural learning at the expense of deep understanding. This shallow acquaintance can also lead to learners feeling insecure[4] and fails to adequately promote a commitment to the subject in students. This approach therefore often leads to apparent success without students developing the depth and tenacity that is needed for long-term progression[10]. In addition, the use of acceleration is in stark contrast to the successful practice in many of the world's mathematically most highly performing jurisdictions.

At the same time it is damaging for young people who have progressed to a notional target to be allowed to coast, repeat work, mark time or otherwise to be under challenged. All young people regardless of their prior attainment should be entitled to a challenging and rewarding classroom mathematics experience.

Ofsted evidence shows that the most effective strategy for generating students' interest in and commitment to mathematics is through planned enrichment and extension work with the minimum of acceleration[9]. The Mathematical Association has also argued the need for the most able students to be routinely expected to master essentially the same material as their peers – but more robustly, fluently and deeply, and with a greater emphasis on making connections[11]. They should also focus on communicating mathematically and on developing better problem solving skills both within and beyond mathematics[1,12]. That is, if students are ultimately to go even further in mathematics, they need to achieve a deeper, more rigorous mastery of core material before moving on.

Instead of national and local policies which seek to identify able mathematicians and then accelerate them through standard material, ACME proposes an approach to mathematics education policy that aims to enrich and deepen the content of their curriculum.

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[1] *Mathematics in the workplace and in Higher Education*, ACME, 2011; **http://www.acme-uk.org/media/7624/acme_theme_a_final%20(2).pdf.**

[2] *A World class Mathematics Education for all*, Conservatives, 2011; **http://www.conservatives.com/News/News_stories/2011/08/~/media/Files/Downloadable%20Files/Vorderman%20maths%20report.ashx**. *House of Lords Select Committee on Science and Technology report*, 2012; **http://www.publications.parliament.uk/pa/ld201213/ldselect/ldsctech/37/37.pdf.** *Making Mathematics Count*, the Report of Professor Adrian Smith's Inquiry into Post-14 Mathematics Education, 2004; **http://www.mathsinquiry.org.uk/report/**

[3] PISA 2009, TIMMS 2007.

[4] Nurturing Able Young Mathematicians, A.Gardiner, 2012; **http://demorgandotorg.files.wordpress.com/2012/02/gardiner_able_10aug12.pdf**

[5] *Educating the highly able,* Sutton Trust, 2012; **http://www.suttontrust.com/research/educating-the-highly-able/**

[6] Examples of the second category include mathematics clubs; participation in the National Mathematics Challenges from Primary upwards; intra-school mathematics competitions; visiting speakers from outside schools; preparation for Olympiads; and enrichment activities to explore mathematics history. The 'beyond school' category ranges from local events organised by universities (e.g. Bath TAPS), or by learned societies (e.g. the Royal Institution masterclasses), or by local industries, or by independent groups (such as MMP, or UKMT, or 'MathsInspiration') to residential Summer Schools.

[7] There will still remain a small number of very exceptional young people who may require bespoke provision. But they are few, and their needs are very varied, so they are not relevant to the formulation of general policy affecting the large majority. Such exceptional provision is not dealt with in this paper.

The National Curriculum provides an outline Programme of Study for the whole cohort aged 5-16. The evidence of the last 30 years or so is that *minimal* provision in line with the National Curriculum is sufficient neither to nurture serious mathematical ability at any age nor as an in-depth preparation for A level mathematics studies. The Programme of Study needs to be 'fleshed out' with exemplification of the depth and rigour needed for well-founded progression. We refer to this as an 'extended interpretation' of the curriculum.

What we propose would, in an ideal world, be provided routinely as part of an expert interpretation of a broad and balanced National Curriculum. Indeed, there are already some excellent schools at both primary and secondary level which do provide quality support for able mathematicians, and lessons should be learned from these[13]. It would supplement and underpin use of the variety of enrichment materials available, enabling their potential to be met in a rigorous way that builds for longterm mathematical success.

Improving access to such a curriculum across a large number of schools is a serious professional challenge: teachers at all phases will need access to a suitable curriculum and assessment structure, and to quality materials – with opportunities to develop the necessary subject knowledge and subject pedagogical knowledge in order to use them effectively. Over time, provision such as that envisaged would become routine in all schools.

Principle 2) Accountability measures should support deep, challenging, rich learning in mathematics

Current accountability policies, from successive governments, while established with the best of motives, have effectively increased 'the class divide'. Many schools concentrate on achieving measures which fail to privilege the kind of solid foundations and attitudes that nurture a disposition for, and future success in, A level Mathematics and beyond, as evidenced in recent reports from the Department for Education[14] and from Ofsted[9].

There is a need in schools for workplace and accountability structures that allow, support and reward time spent on extension and enrichment of the curriculum. Accountability and assessment structures must measure the depth of students' understanding, and not reward the acceleration approach discussed earlier in this paper.

At primary level, deep and extended understanding of the Key Stage 1 and 2 mathematics curriculum should be valued by Government, schools, parents and pupils rather than a superficial acquaintance with material from Key Stage 3. The current provision of level 6 tests at Key stage 2 has the potential to drive acceleration of pupils, and hinder a secure understanding of Key Stage 2 mathematics. Accountability measures for the new primary curriculum should not include any measures of attainment at Key Stage 3.

At Key Stage 4, ACME has argued in the past against schools' propensity to enter students early to GCSE Mathematics, whether in an attempt to accelerate high achieving students or for other reasons.[15] This early entry to standard assessments is often not in the best interests of the student – a position that is now acknowledged and supported by Ofsted. The sitting of the new EBCs should be more of an integrated part of the total mathematics experience for students, rather than something that follows years of specific preparation for the tests.

Principle 3) Rich curriculum provision should precede identification of the most able

An important characteristic of practice in the most successful schools is that identification of able mathematicians *follows* rich curriculum provision in schools. An 'identify and treat' model is not appropriate, given the variety of characteristics and development paths of able young mathematicians.

The identification of potential should be through teachers' observations of young people's responses to a rich and challenging classroom experience. We envisage that the extended curriculum provision called for here would be available to around 30% of the cohort, and that approximately 20% of the overall cohort would significantly master this material.

Informed, skilled and sensitive teaching is vital. Teachers need to be alert to opportunities to develop rigour in mathematics learning. They also need to be alert to the ability of those students who are able to make links and connections between mathematical concepts.

The development of mathematical talent is a long-term process that is dependent on many variables, including quality teaching and a student's attitude to the subject. Mastery of core skills and knowledge is necessary for good mathematical progression and should not be undervalued, but developed in conjunction with the range of valued mathematical behaviours in a progressively deep and rigorous way[12].

The identification of mathematically able students should take into account the diversity of children's progression and learning styles – young people learn in different ways and at different rates. Identification of particular potential should be as a result of response to the provision of a challenging and engaging curriculum for all students.

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[8] *Mathematical Needs*, ACME, 2011; **http://www.acme-uk.org/media/7630/acme_4pp_overarching_report_summary.pdf**

[9] *Made to Measure*, Ofsted, 2012; **http://www.ofsted.gov.uk/resources/mathematics-made-measure**

[10] *Understanding the Score*, Ofsted 2009.

[11] **www.m-a.org.uk/resources/Advice%20on%20Most%20Able%20Secondary.doc**

[12] *The Mathematical Needs of the Learners,* ACME, 2011; **http://www.acme-uk.org/media/7627/acme_theme_b_final.pdf**

[13] *Good Practice in Primary Mathematics*, Ofsted, 2011, **http://www.ofsted.gov.uk/resources/good-practice-primary-mathematics-evidence-20-successful-schools**, and *Good practice resource - Engaging able mathematics students: King Edward VI Camp Hill School for Boys*, Ofsted 2012, **http://www.ofsted.gov.uk/resources/good-practice-resource-engaging-able-mathematics-students-king-edward-vi-camp-hill-school-for-boys**

[14] *Early Entry to GCSE Examinations*, DfE, 2011; **https://www.education.gov.uk/publications/eOrderingDownload/Early-entries-GCSEs.pdf**

[15] *Position Paper on Early and Multiple Entry to GCSE Mathematics*, ACME, 2011 **http://www.acme-uk.org/news/news-items-repository/2011/5/position-paper-on-early-and-mutiple-entry-to-gcse-mathematics**

[16] Rethinking Maths for the 21st Century, Cambridge Network; **http://www.cambridgenetwork.co.uk/news/rethinking-maths-for-the-21st-century/**

Recommendations

Recommendation 1:
Providing an extended interpretation of the mathematics curriculum

The National Curriculum should be enhanced with an 'extended interpretation' of the listed content to demonstrate the greater *depth, challenge and sophistication of link-making* envisaged. This extended interpretation of the mathematics curriculum would be a more challenging version of core curriculum content, though it might incorporate a small amount of additional material. There will be young people in every mainstream school who would benefit from being exposed to this mathematical experience.

The design, development and implementation of this interpretation of the curriculum should be properly funded, piloted and devolved to a small group of knowledgeable people from the mathematics and mathematics education communities. The group should have a remit including an obligation to consult with the relevant stakeholders.This group would work closely with any panel having oversight of the wider mathematics curriculum and assessment systems. Such provision should form part of the non-statutory guidance for Key Stages 1 to 4.

The curriculum for *all* students should build on the National Curriculum to give significant weight to the development of problem-solving and reasoning, both of which are critical to the development of mathematical potential.

Recommendation 2:
Access to the extended mathematics curriculum

We envisage that around 30% of the cohort will find the extended interpretation a challenging but satisfying experience: access should be made available in all mainstream schools. The extended interpretation should be implemented concurrently with the new National Curriculum: this issue is both serious and urgent.

Recommendation 3:
Assessment of the curriculum

Formal assessment of the extended interpretation should be incorporated into assessment structures at ages 11 and 16: it might eventually be targeted at 20-25% of any cohort. We recommend that this extended interpretation is the basis for the Additional Mathematics assessment at age 16, as proposed in the current consultation on English Baccalaureate Certificate. Some exposure to material in the extended interpretation of Key Stage 4 should be a prerequisite for A level Mathematics.

Recommendation 4:
Initial Teacher Education and CPD should support implementation

Taking this approach to developing the most able will enhance the teaching of mathematics for *all* young people. Teachers will need support to develop their skills in these areas, given the legacy of widespread lack of deep rigour in school mathematics over the years. Training should complement other subject-specific CPD, for example that required to implement the new National Curriculum.

ITE for all primary and all secondary mathematics teachers should address the skills and knowledge necessary to teach the level of mastery and link-making envisioned in the extended interpretation of the curriculum. Those working in ITE urgently need access to substantial funded CPD.

Over a period of say five years, specifically designed, subject specific CPD should be made freely available to all teachers of mathematics. This should focus (i) on the scope for key material to be mastered in greater depth; (ii) on connections between topics; and (iii) on using investigative work and problem solving to help pupils learn mathematics, together with the appropriate related pedagogy.

Recommendation 5:
Develop support materials

The principles above should be fully reflected in core curriculum materials. However, in the short term there is a need for central funding, and support for the development of suitable materials (including 'textbooks' and online resources).

A substantial new curriculum initiative at A level[16] was recently launched and similar large-scale projects could be developed for primary and secondary. This would allow teachers to develop the requisite additional subject and subject pedagogical knowledge that underpin the extended interpretation, and to construct an effective pedagogical model. Textbook and curriculum resource development should take place in coordination with other moves towards high-quality textbooks, to ensure consistency and progression between all materials.

Recommendation 6:
Develop appropriate accountability measures

Accountability and inspection measures need to recognise the fragility of good provision in this area. Suitable measures include checking provision for and access to extended interpretation mathematics material, and progression rates to A level Mathematics and Further Mathematics at 16. Accountability measures for one Key Stage should not include any incentive for schools to encourage students to progress prematurely to the next Key Stage.

It is important to assess the quality of the range of enriched classroom provision, whose full benefits might take years to become apparent, and cannot be measured solely by student performance in external examinations. Such provision is not easy to assess in an inspection – especially by a non-specialist; those who are required to make such judgments will also need a degree of professional support. We should work towards a position where formal inspection of all mathematics teaching is by specialists who know how to observe the conceptual development and coherence of mathematics lessons.

Recommendation 7: Beyond the classroom

Although appropriate in-class provision is the most urgent need for able young mathematicians, extra-curricular provision as outlined in footnote 6 is also important for nurturing robust mathematical inclinations. Ways of encouraging primary and secondary schools to develop and maintain such wider participation (such as overt valuing of them in general Ofsted inspections, and an extension of the grants for schemes listed in the STEM Directory) should be adopted.

Recommendation 8: Cross-party support

Effective development of our able young mathematicians, and the achievement of national economic and social goals, requires a sustained and consistent strategy. Serious attempts should therefore be made to reach a substantial degree of cross-party support for the development of this provision.

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